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A three-dimensional inverse problem in imaging the local heat transfer coefficients for plate finned-tube heat exchangers

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Abstract

A three-dimensional inverse heat conduction problem in imaging the local heat transfer coefficients for plate finned-tube heat exchangers utilizing the steepest descent method and a general purpose commercial code CFX4.4 is applied successfully in the present study based on the simulated measured temperature distributions on fin surface by infrared thermography.

It is assumed that no prior information is available on the functional form of the unknown local heat transfer coefficients in the present study. Thus, it can be classified as function estimation for the inverse calculations.

Two different heat transfer coefficients for in-line tube arrangements with different measurement errors are to be estimated. Results show that the present algorithm can obtain the reliable estimated heat transfer coefficients.

1. Introduction

Finned surfaces of the plate finned-tube heat exchangers have been in use over a long period of time for dissipation of heat by convection. Applications for finned surfaces are widely seen in air-conditioning, electrical, chemical, refrigeration, cryogenics and many cooling systems in industrial. Kays and London [1] introduced various types of heat transfer surfaces.

For the purpose of energy savings, it is important to design better heat transfer surfaces such that high efficient heat transfer equipment can be obtained. To achieve this goal the estimation of local convective heat transfer coefficients for fin surface becomes very important in designing the high-performance heat exchangers. However, the estimation of the convective heat transfer coefficient is more difficult to perform than other common thermo-fluid-dynamic quantities, especially in case of non-uniform distributions and/or of conduction–convection problem.

Recently, Ay et al. [2] used the technique of energy balance based on infrared thermography to estimate the local heat transfer coefficients of plate fin in a 2-D inverse heat conduction problem. The 3-D inverse heat conduction problem applied in estimating the local convective heat transfer coefficients on fin surface has never been seen in the literature.

The commercial codes can be used to calculate many practical but difficult direct thermal problems. If one can device an inverse algorithm, which has the ability to communicate with those commercial codes by means of data transportation, a generalized 3-D inverse heat conduction problem for plate finned-tube heat exchangers can thus be established and used to estimate the local convective heat transfer coefficient.

The technique of combining the inverse algorithms and commercial code CFX4.4 [3] has been developed by Huang and Wang [4] and applied to a 3-D inverse heat conduction problem in estimating the unknown surface heat flux. Based on the similar algorithm Haung and Chen [5] estimated successfully the unknown boundary heat flux in a 3-D inverse heat convection problem.

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Nomenclature

$J[h(S_i)]$	functional defined by Eq. (3)
$J'[h(S_i)]$	gradient of functional defined by Eq. (15)
k	thermal conductivity
$P^n(S_i)$	direction of descent defined by Eq. (5)
$T(\Omega)$	estimated temperature
$\Delta T(\Omega)$	sensitivity function defined by Eq. (6)
$Y(S_i)$	measured temperature
Greek symbols	
β	search step size
$\lambda(\Omega)$	Lagrange multiplier defined by Eq. (12)
ε	convergence criteria
Superscripts	
n	iteration index
$\wedge$	estimated value

Huang and Cheng [6] estimated the heat generation rate of chips on a PC board. More recently, Huang and Lee [7] applied the algorithm to a 3-D optimal control problem. We should note that all of those applications are 3-D problems, this implies that the algorithm is powerful since the three-dimensional inverse problems are still limited in the open literature.

The objective of the present study is to utilize the technique of steepest descent method (SDM) [8] together with commercial code CFX4.4 in estimating local convective heat transfer coefficients of finned surfaces for the 3-D plate finned-tube heat exchangers based on the simulated temperature measurements by infrared thermography.

The SDM has great potential in solving three-dimensional inverse problem. It derives basis from the perturbational principle [8] and transforms the inverse problem to the solution of three problems, namely, the direct problem, the sensitivity problem and the adjoint problem, which will be discussed in detail in the text.

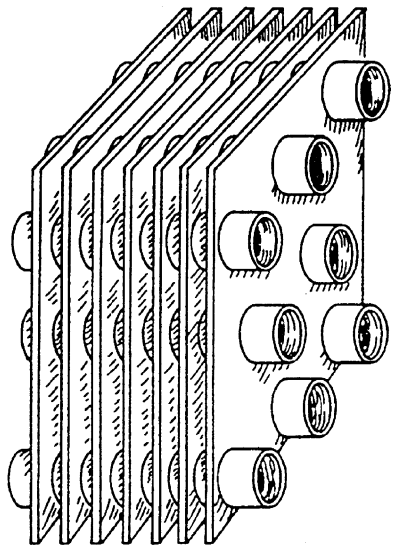

Fig. 1. A typical plate finned-tube heat exchanger.

2. Direct problem

To develop the methodology for use in determining the local convective heat transfer coefficients on the fin surface, we consider the following three-dimensional inverse heat conduction problem. A typical plate finned-tube heat exchanger is shown in Fig. 1. The plate fin with domain $\Omega(x,y,z)$ is illustrated in Fig. 2a, the boundary surface on S_i, $i = 1$–6, are subjected to a convective condition with prescribed heat transfer coefficient $h(S_i)$, $i = 1$–6, where $i = 1$–4 represent the edge boundaries while $i = 5$ and 6 indicate the top and bottom surfaces, respectively. The heat transfer coefficient $h(S_i)$ could be function of temperature in the present study. The tube boundary surfaces on S_i, $i = 7$–15, are subjected to a prescribed temperature condition $T = T(S_i)$, $i = 7$–15. Here k is the thermal conductivity of fin.

The formulation of this three-dimensional steady-state heat conduction problem can be expressed as

$$\frac{\partial^2 T(\Omega)}{\partial x^2} + \frac{\partial^2 T(\Omega)}{\partial y^2} + \frac{\partial^2 T(\Omega)}{\partial z^2} = 0; \quad \text{in } \Omega(x,y,z) \tag{1a}$$

$$-k\frac{\partial T(S_1)}{\partial x} = h(S_1)(T_\infty - T); \quad \text{on fin surface } S_1 \tag{1b}$$

$$-k\frac{\partial T(S_2)}{\partial x} = h(S_2)(T - T_\infty); \quad \text{on fin surface } S_2 \tag{1c}$$

$$-k\frac{\partial T(S_3)}{\partial y} = h(S_3)(T_\infty - T); \quad \text{on fin surface } S_3 \tag{1d}$$

$$-k\frac{\partial T(S_4)}{\partial y} = h(S_4)(T - T_\infty); \quad \text{on fin surface } S_4 \tag{1e}$$

$$-k\frac{\partial T(S_5)}{\partial z} = h(S_5)(T_\infty - T); \quad \text{on fin surface } S_5 \tag{1f}$$

$$-k\frac{\partial T(S_6)}{\partial z} = h(S_6)(T - T_\infty); \quad \text{on fin surface } S_6 \tag{1g}$$

$$T = T(S_i); \quad \text{on tube surfaces, } i = 7\text{–}15 \tag{1h}$$

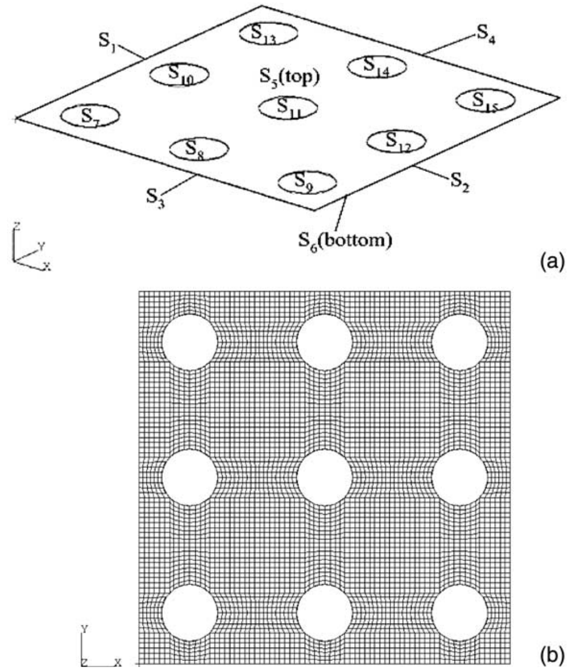

Fig. 2. (a) The geometry of plate fin for the present study. (b) The grid system for the present study.

Due to the nature that the edge surface area S_i, $i = 1$–4 is small enough when comparing with top and bottom surfaces S_i, $i = 5$–6. This also implies that the heat transfer rate through S_i, $i = 1$–4 is rather smaller than S_i, $i = 5$–6. For this reason we assumed that the boundary conditions on surface S_i, $i = 1$–4 are insulated. Moreover, the fin thickness is very thin, so the temperature distribution on S_5 should be very close to S_6, for this reason it is also reasonable to assumed the heat transfer coefficients on S_5 equal to S_6, i.e. $h(S_5) = h(S_6)$. The direct problem becomes

$$\frac{\partial^2 T(\Omega)}{\partial x^2} + \frac{\partial^2 T(\Omega)}{\partial y^2} + \frac{\partial^2 T(\Omega)}{\partial z^2} = 0; \quad \text{in } \Omega(x,y,z) \tag{2a}$$

$$\frac{\partial T(S_i)}{\partial n} = 0; \quad \text{on fin surface } S_i,\ i = 1\text{–}4 \tag{2b}$$

$$-k\frac{\partial T(S_5)}{\partial z} = h(S_5)(T_\infty - T); \quad \text{on fin surface } S_5 \tag{2c}$$

$$-k\frac{\partial T(S_6)}{\partial z} = h(S_6)(T - T_\infty); \quad \text{on fin surface } S_6 \tag{2d}$$

$$T = T(S_i); \quad \text{on tube surfaces},\ i = 7\text{–}15 \tag{2e}$$

The direct problem considered here is concerned with calculating the plate fin temperatures when the heat transfer coefficient $h(S_i)$, $i = 5$ and 6, thermal conductivity and boundary condition on tube surfaces are known. The solution for the above 3-D heat conduction problem in domain Ω is solved using CFX4.4 and it's Fortran subroutine USRBCS.

3. The inverse problem

For the inverse problem, the local heat transfer coefficient $h(S_i)$, $i = 5$ and 6, is regarded as being unknown, but everything else in Eq. (2) is known. In addition, the simulated temperature readings using infrared thermography on the fin surface S_5 and S_6 are assumed available.

Let the temperature reading taken by infrared scanners on fin surfaces S_5 and S_6 be denoted by $Y(S_i) \equiv Y(x_m, y_m) \equiv Y_m(S_i)$, $m = 1$–M and $i = 5$ and 6, where M represents the number of measured temperature extracting points. We note that the measured temperature $Y_m(S_i)$ contain measurement errors. Then the inverse problem can be stated as follows: by utilizing the above mentioned measured temperature data $Y_m(S_i)$, estimate the unknown local heat transfer coefficient $h(S_i)$, $i = 5$ and 6.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$J[h(S_i)] = \sum_{m=1}^{M} [T_m(S_i) - Y_m(S_i)]^2; \quad i = 5 \text{ and } 6 \tag{3}$$

here, $T_m(S_i)$ are the estimated or computed temperatures at the measured temperature extracting locations (x_m, y_m). These quantities are determined from the solution of the direct problem given previously by using the estimated local heat transfer coefficient $h(S_i)$.

4. Steepest destent method for minimization

The following iterative process based on the SDM [8] is now used for the estimation of unknown heat transfer coefficient $h(S_i)$ by minimizing the functional $J[h(S_i)]$

$$h^{n+1}(S_i) = h^n(S_i) - \beta^n P^n(S_i); \quad \text{for } n = 0, 1, 2, \ldots \text{ and } i = 5, 6 \tag{4}$$

where β^n is the search step size in going from iteration n to iteration $n + 1$, and $P^n(S_i)$ is the direction of descent (i.e. search direction) given by

$$P^n(S_i) = J'^n(S_i); \quad i = 5 \text{ and } 6 \tag{5}$$

which is identical to the gradient direction $J'^n(S_i)$ at iteration n.

To perform the iterations according to Eq. (4), we need to compute the step size β^n and the gradient of the functional $J'^n(S_i)$. In order to develop expressions for the determination of these two quantities, a ‘‘sensitivity problem" and an "adjoint problem" are constructed as described below.

4.1. Sensitivity problem and search step size

It is assumed that when $h(S_i)$ undergoes a variation Δh, T is perturbed by $T+\Delta T$. Then replacing in the direct problem h by $h+\Delta h$ and T by $T+\Delta T$, subtracting from the resulting expressions the direct problem and neglecting the second-order terms, the following sensitivity problem for the sensitivity function ΔT are obtained:

$$\frac{\partial^2 \Delta T(\Omega)}{\partial x^2}+\frac{\partial^2 \Delta T(\Omega)}{\partial y^2}+\frac{\partial^2 \Delta T(\Omega)}{\partial z^2}=0; \quad \text{in } \Omega(x,y,z) \tag{6a}$$

$$\frac{\partial \Delta T(S_i)}{\partial n}=0; \quad \text{on fin surfaces } S_i,\ i=1\text{–}4 \tag{6b}$$

$$-h\,\Delta T+k\frac{\partial \Delta T}{\partial z}=\Delta h(T-T_\infty); \quad \text{on fin surface } S_5 \tag{6c}$$

$$h\,\Delta T+k\frac{\partial \Delta T}{\partial z}=\Delta h(T_\infty-T); \quad \text{on fin surface } S_6 \tag{6d}$$

$$\Delta T=0; \quad \text{on tube surfaces},\ i=7\text{–}15 \tag{6e}$$

CFX 4.4 is used to solve above sensitivity problem.

The functional $J[h^{n+1}(S_i)]$ for iteration $n+1$ is obtained by rewriting Eq. (3) as

$$J[h^{n+1}(S_i)]=\sum_{m=1}^{M}[T_m(S_i;h^n-\beta^n P^n)-Y_m(S_i)]^2; \quad i=5 \text{ and } 6 \tag{7}$$

where we replaced h^{n+1} by the expression given by Eq. (4). If temperature $T_m(h^n-\beta^n P^n)$ is linearized by a Taylor expansion, Eq. (7) takes the form

$$J[h^{n+1}(S_i)]=\sum_{m=1}^{M}[T_m(S_i;h^n)-\beta^n\,\Delta T_m(S_i;P^n)-Y_m(S_i)]^2; \quad i=5 \text{ and } 6 \tag{8}$$

where $T_m(S_i;h^n)$ is the solution of the direct problem by using estimate heat transfer coefficient for exact heat transfer coefficient on S_i, $i=5$ and 6. The sensitivity functions $\Delta T_m(S_i;P^n)$ are taken as the solutions of problem (6) at the measured temperature extracting positions (x_m,y_m,z_m) by letting $\Delta h=P^n$. The search step size β^n is determined by minimizing the functional given by Eq. (8) with respect to β^n. The following expression results:

$$\beta^n=\frac{\sum_{m=1}^{M}[T_m(S_i)-Y_m(S_i)]\Delta T_m(S_i)}{\sum_{m=1}^{M}[\Delta T_m(S_i)]^2}; \quad i=5 \text{ and } 6 \tag{9}$$

4.2. Adjoint problem and gradient equation

To obtain the adjoint problem, Eq. (2a) is multiplied by the Lagrange multiplier (or adjoint function) $\lambda(\Omega)$ and the resulting expression is integrated over the correspondent space domain. Then the result is added to the right hand side of Eq. (3) to yield the following expression for the functional $J[h(S_5)]$:

$$\begin{aligned} J[h(S_i)] &= \sum_{m=1}^{M}[T_m(S_i)-Y_m(S_i)]^2+\int_\Omega[\lambda(\Omega)\times\nabla^2 T]\,\mathrm{d}\Omega \\ &= \int_{S_i}[T(S_i)-Y(S_i)]^2\delta(x-x_m)\delta(y-y_m)\,\mathrm{d}S_i \\ &\quad +\int_\Omega[\lambda(\Omega)\times\nabla^2 T]\,\mathrm{d}\Omega \\ &\text{in } \Omega(x,y,z),\ i=5 \text{ and } 6 \end{aligned} \tag{10}$$

The variation ΔJ can be obtained by perturbing h by Δh and T by ΔT in Eq. (10), subtracting from the resulting expression the original Eq. (10) and neglecting the second-order terms. We thus find

$$\begin{aligned} \Delta J[h(S_i)] &= \int_{S_i} 2[T(S_i)-Y(S_i)]\Delta T(S_i)\delta(x-x_m) \\ &\quad\times\delta(y-y_m)\,\mathrm{d}S_i+\int_\Omega[\lambda(\Omega)\times\nabla^2\Delta T]\,\mathrm{d}\Omega \\ &\text{in } \Omega(x,y,z),\ i=5 \text{ and } 6 \end{aligned} \tag{11}$$

where $\delta(\cdot)$ is the Dirac delta function and (x_m,y_m), $m=1$–M, refers to the measured temperature extracting positions. In Eq. (11), the domain integral term is reformulated based on the Green's second identity; the boundary conditions of the sensitivity problem given by Eqs. (6b)–(6e) are utilized and then ΔJ is allowed to go to zero. The vanishing of the integrands containing ΔT leads to the following adjoint problem for the determination of $\lambda(\Omega)$:

$$\frac{\partial^2 \lambda(\Omega)}{\partial x^2}+\frac{\partial^2 \lambda(\Omega)}{\partial y^2}+\frac{\partial^2 \lambda(\Omega)}{\partial z^2}=0; \quad \text{in } \Omega(x,y,z) \tag{12a}$$

$$\frac{\partial \lambda(\Omega)}{\partial n}=0; \quad \text{on fin surfaces } S_i,\ i=1\text{–}4 \tag{12b}$$

$$-\lambda h+k\frac{\partial\lambda}{\partial n}=2k[T(S_5)-Y(S_5)]\delta(x-x_m)\delta(y-y_m); \quad \text{on fin surface } S_5 \tag{12c}$$

$$\lambda h+k\frac{\partial\lambda}{\partial n}=2k[T(S_6)-Y(S_6)]\delta(x-x_m)\delta(y-y_m); \quad \text{on fin surface } S_6 \tag{12d}$$

$$\lambda=0; \quad \text{on tube surfaces},\ i=7\text{–}15 \tag{12e}$$

Finally, the following integral term is left:

$$\Delta J = \int_{S_5} \frac{\lambda(S_5)}{k}[T(S_5) - T_\infty]\Delta h(S_5)\,\mathrm{d}S_5 - \int_{S_6} \frac{\lambda(S_6)}{k}[T(S_6) - T_\infty]\Delta h(S_6)\,\mathrm{d}S_6 \quad (13)$$

From definition [9], the functional increment can be presented as

$$\Delta J = \int_{S_5} J'(S_5)\Delta h(S_5)\,\mathrm{d}S_5 - \int_{S_6} J'(S_6)\Delta h(S_6)\,\mathrm{d}S_6 \quad (14)$$

A comparison of Eqs. (13) and (14) leads to the following expression for the gradient of the functional $J[h(S_i)]$:

$$J'[h(S_i)] = \frac{\lambda(S_i)}{k}[T(S_i) - T_\infty]; \quad \text{on surfaces } S_i, \quad i = 5 \text{ and } 6 \quad (15)$$

We note that based on Eq. (12e) the gradient J' on tube surface is always equal to zero. If the initial guess values of h^0 cannot be predicted correctly before the inverse calculation, the estimated values of heat transfer coefficients will deviate from exact values near the tube surface. This is the case in the present study!

4.3. Stopping criterion

If the problem contains no measurement errors, the traditional check condition is specified as

$$J[h^{n+1}(S_i)] < \varepsilon; \quad i = 5 \text{ and } 6 \quad (16)$$

where ε is a small-specified number. However, the observed temperature data may contain measurement errors. Therefore, we do not expect the functional equation (3) to be equal to zero at the final iteration step. Following the experiences of the authors [2,4–6,9], we used the discrepancy principle as the stopping criterion, i.e. we assume that the temperature residuals may be approximated by

$$T_m(S_i) - Y_m(S_i) \approx \sigma; \quad i = 5 \text{ and } 6 \quad (17)$$

where σ is the standard deviation of the measurements, which is assumed to be a constant. Substituting Eq. (17) into Eq. (3), the following expression is obtained for stopping criteria ε:

$$\varepsilon = 2M\sigma^2 \quad (18)$$

Then, the stopping criterion is given by Eq. (16) with ε determined from Eq. (18).

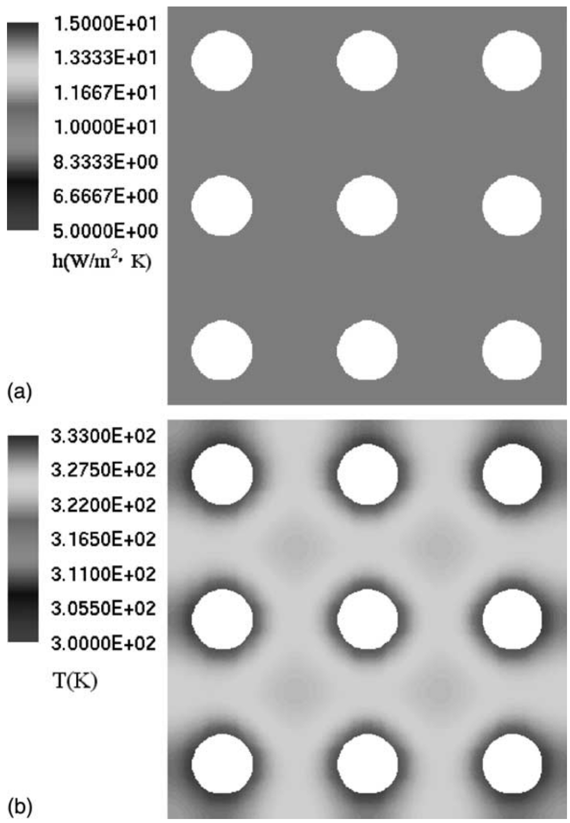

Fig. 3. The exact value for (a) heat transfer coefficients and (b) measured temperatures in test case 1.

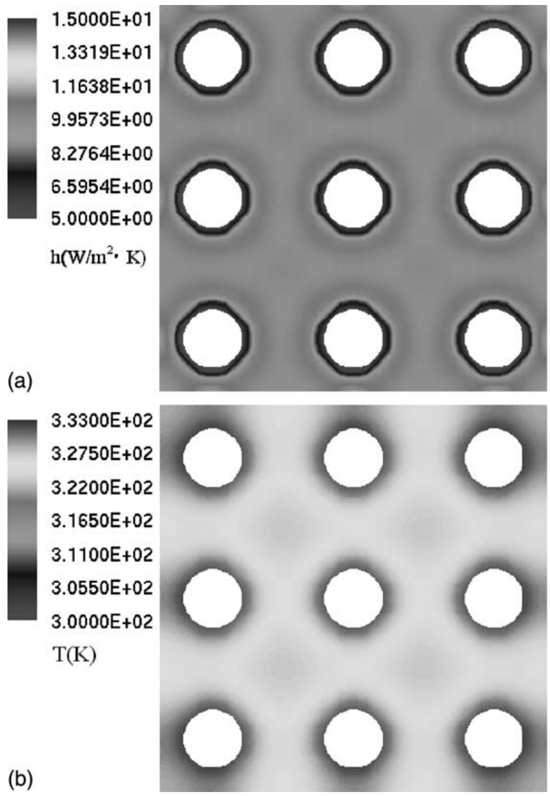

Fig. 4. The estimated value with $\sigma = 0.0$ for (a) heat transfer coefficients and (b) measured temperatures in test case 1.

5. Computational procedure

The computational procedure for the solution of this inverse problem using SDM may be summarized as follows:

Suppose $h^n(S_i)$ is available at iteration n.

Step 1. Solve the direct problem given by Eq. (2) for $T(\Omega)$.
Step 2. Examine the stopping criterion given by Eq. (16) with ε given by Eq. (18). Continue if not satisfied.
Step 3. Solve the adjoint problem given by Eq. (12) for $\lambda(\Omega)$.
Step 4. Compute the gradient of the functional J' from Eq. (15).
Step 5. Compute the direction of descent P^n from Eq. (5).
Step 6. Set $\Delta h = P^n$, and solve the sensitivity problem given by Eq. (6) for $\Delta T(\Omega)$.
Step 7. Compute the search step size β^n from Eq. (9).
Step 8. Compute the new estimation for h^{n+1} from Eq. (4) and return to step 1.

6. Results and discussions

The objective of the present study is to show the validity of the SDM in estimating the local surface heat transfer coefficients for a three-dimensional plate finned-tube heat exchangers with no prior information on the functional form of the unknown function. The physical model for this problem is described as follows: The thermal conductivity for plate fin is taken as $k = 43$ W/(m^2 K), ambient temperature is chosen as $T_\infty = 300$ K and the temperatures on tube surface are assumed as $T(S_i) = 333$ K, $i = 7$–16.

To illustrate the ability of the SDM in predicting $h(S_i)$, $i = 5$ and 6, with inverse analysis from the knowledge of the simulated measured temperature distributions on fin surface, we consider two numerical test cases with different variation of $h(S_i)$.

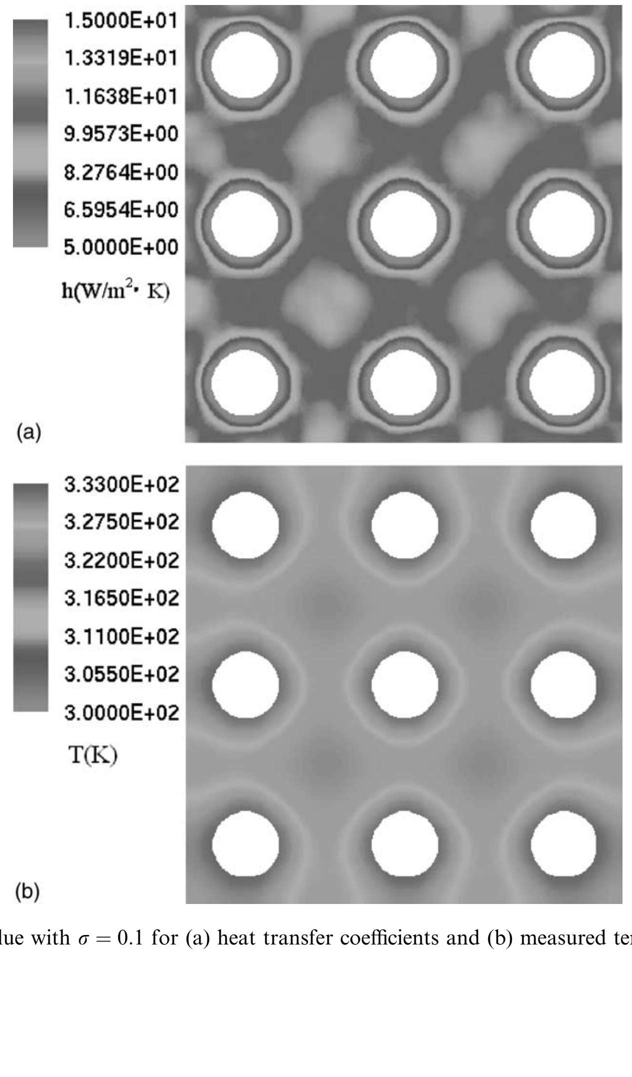

Fig. 5. The estimated value with $\sigma = 0.1$ for (a) heat transfer coefficients and (b) measured temperatures in test case 1.

One of the advantages of using the SDM is that the initial guesses of the unknown heat transfer coefficients $h(S_i)$ can be chosen arbitrarily. In all the test cases considered here, the initial guesses of heat transfer coefficients used to begin the iteration are taken as $h(S_i) = 0.0$.

In order to compare the results for situations involving random measurement errors, we assume normally distributed uncorrelated errors with zero mean and constant standard deviation. The simulated inexact measurement data Y can be expressed as

$$Y_m = Y_{m,\text{exact}} + \omega\sigma \tag{19}$$

where $Y_{m,\text{exact}}$ is the solution of the direct problem with an exact heat transfer coefficients; σ is the standard deviation of the measurements; and ω is a random variable that generated by subroutine DRNNOR of the IMSL [10] and will be within −2.576 to 2.576 for a 99% confidence bound.

We now present below the numerical experiments in determining $h(S_i)$ by the inverse analysis.

6.1. Numerical test case 1

The geometry for the first test case is shown in Fig. 2a, which represents an in-lined tube arrangement for a fin plate. The dimension for fin in x-, y- and z-directions is 167, 167 and 1 mm, respectively. The radius of tube is taken as 12.7 mm and the longitudinal pitch of tube (i.e. the distance between center of two tubes) is 60.7 mm. The grid system for the present study is shown in Fig. 2b. The grid in z-direction is taken as 5 and the total grid number on x–y plane is 1800, therefore there are totally 9000 grids in the present study.

The exact surface heat transfer coefficients is assumed as constant on S_5 and S_6 and are taken as $h(S_5) = h(S_6) = 10$ W/(m^2 K) in test case 1.

The three-dimensional inverse problem is first examined by using exact measurements, i.e. $\sigma = 0.0$. The

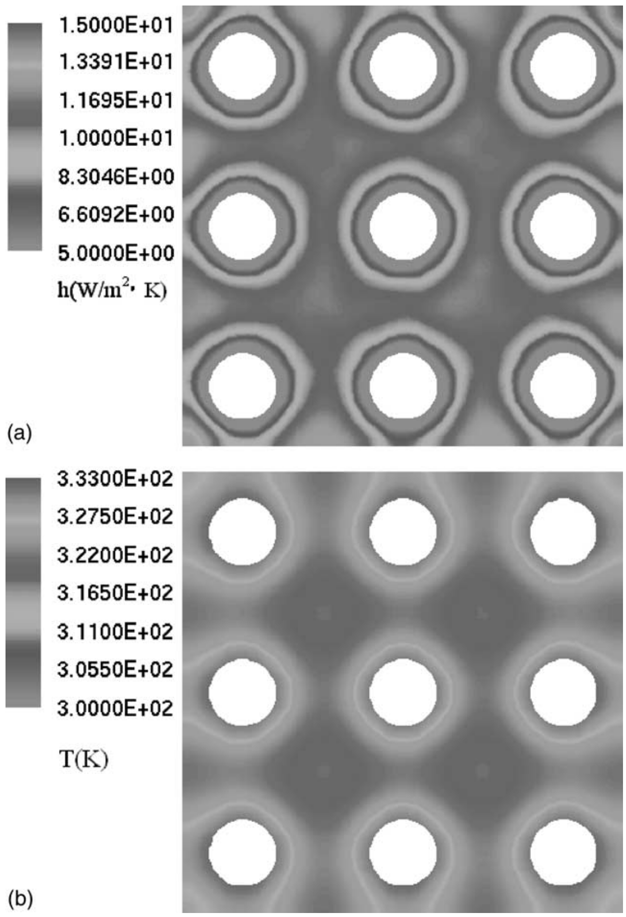

Fig. 6. The estimated value with $\sigma = 0.4$ for (a) heat transfer coefficients and (b) measured temperatures in test case 1.

exact heat transfer coefficients $h(S_5)$ and exact temperature measurements $Y(S_5)$ are shown in Fig. 3a and b, respectively.

By setting stopping criteria $\varepsilon = 0.01$, after 42 iterations the inverse solutions converged. The estimated heat transfer coefficients and estimated (or calculated) fin surface temperature are shown in Fig. 4a and b, respectively.

By comparing those figures we find that the estimated temperatures are almost identical to the measured temperatures since the relative error between these two temperatures is calculated as $\mathrm{ERR1} = 0.03\%$, where ERR1 is defined as

$$\mathrm{ERR1}\% = \left[\sum_{m=1}^{M}\left|\frac{T_m(S_5) - Y_m(S_5)}{Y_m(S_5)}\right|\right] \Big/ (M) \times 100\% \tag{20}$$

here M represents the number of grids.

The estimated heat transfer coefficients are also very close to the exact value except for the position near tube surface. This is due to the singularity of the gradient equation on tube surface as was discussed previously. The accuracy of estimated heat transfer coefficients near tube surface can be improved by increasing the number of iterations, however, CPU time for the calculations will also be increased.

The relative error between exact and estimated heat transfer coefficients is calculated as $\mathrm{ERR2} = 2.15\%$, where ERR2 is defined as

$$\mathrm{ERR2}\% = \left[\sum_{m=1}^{M}\left|\frac{h_m(S_i) - \hat{h}_m(S_i)}{h_m(S_i)}\right|\right] \Big/ (M) \times 100\% \tag{21}$$

here M represents the number of grids and $\hat{h}_m(S_i)$ indicates the estimated values.

The inverse calculation is then proceed to consider the inexact temperature measurements. The standard deviation of the measurements is first taken as $\sigma = 0.1$, then it was increased to $\sigma = 0.4$.

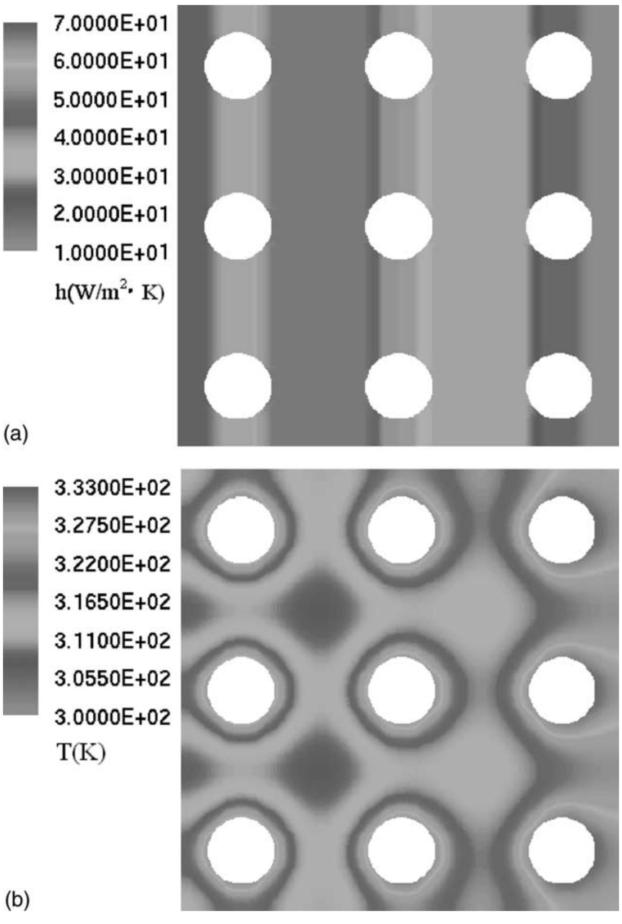

Fig. 7. The exact value for (a) heat transfer coefficients and (b) measured temperatures in test case 2.

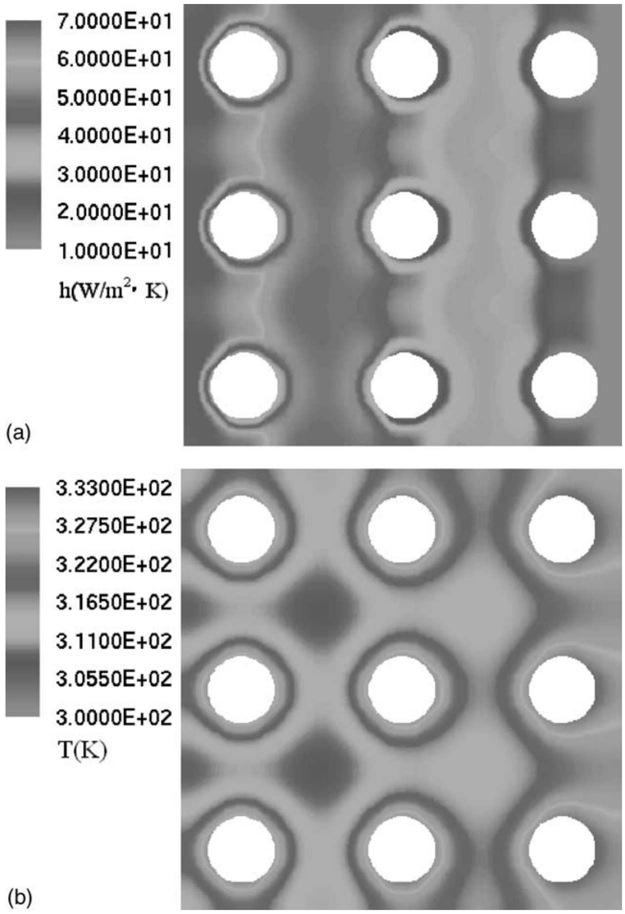

Fig. 8. The estimated value with $\sigma = 0.0$ for (a) heat transfer coefficients and (b) measured temperatures in test case 2.

For $\sigma = 0.1$, 20 iterations are needed to satisfy the stopping criteria based on the discrepancy principle, the estimated heat transfer coefficients and temperatures are shown in Fig. 5a and b, respectively. The relative errors for temperatures and heat transfer coefficients are calculated as $\mathrm{ERR1} = 0.23\%$ and $\mathrm{ERR2} = 5.92\%$. For $\sigma = 0.4$, the number of iterations to satisfy the stopping criteria is only 8, the estimated heat transfer coefficients and temperatures are shown in Fig. 6a and b, respectively, and the relative errors for temperatures and heat transfer coefficients are calculated as $\mathrm{ERR1} = 2.51\%$ and $\mathrm{ERR2} = 11.72\%$.

6.2. Numerical test case 2

In order to show the potential of the present algorithm for use in a three-dimensional inverse problem, we consider the second numerical test case. The geometry and grid systems of this case are the same as used in test case 1.

The exact distribution for heat transfer coefficients is now assumed varying linearly on S_5 from 10 to 70 W/(m^2 K) as shown in Fig. 7a, while Fig. 7b represents the exact temperature measurements.

When assuming $\sigma = 0.0$ and setting $\varepsilon = 0.01$, after 45 iterations the estimated heat transfer coefficients can be obtained. The estimated heat transfer coefficients and temperatures are shown in Fig. 8a and b, respectively. It is obvious that the estimated values around tube surface are slightly deviated from the exact values, however the overall estimations are still reliable. ERR1 and ERR2 are calculated as 0.09% and 3.75%, respectively.

From above two numerical test cases we concluded that the SDM is now applied successfully in this three-dimensional inverse heat conduction problem for predicting the surface heat transfer coefficients of plate fin.

7. Conclusions

The SDM with adjoint equation was successfully applied for the solution of a three-dimensional inverse heat conduction problem in determining the local heat transfer coefficients for plate finned-tube heat exchangers. Two test cases involving different type of heat transfer coefficients and different measurement errors were considered. The results show that the SDM does not require a priori information for the functional form of the unknown functions and the reliable estimated values can always be obtained.

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